Eur. Phys. J. C 11, 501–506 (1999)
Digital Object Identifier (DOI) 10.1007/s100529900952

THE EUROPEAN PHYSICAL JOURNAL C

Twist-3 contribution to the pion electromagnetic form factor

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Received: 23 March 1998 / Published online: 3 November 1999

Abstract. Non-leading contribution to the pion electromagnetic form factor which comes from the pion twist-3 wave function is analyzed in the modified hard scattering approach (MHSA) proposed by Li and Sterman. This contribution is enhanced significantly due to bound state effect (the twist-3 wave function is independent of the fractional momentum carried by the parton and has a large factor $\sim m_\pi^2/m_0$ with m_π being the pion meson mass and m_0 being the mean u- and d-quark masses). Consequently, although it is suppressed by the factor $1/Q^2$, the twist-3 contribution is comparable with and even larger than the leading twist (twist-2) contribution at intermediate energy region of Q^2 being $2 \sim 40\,\mathrm{GeV}^2$.

1 Introduction

There has been a lot of discussions about applying perturbative QCD (pQCD) to exclusive processes at large momentum transfer [1–15]. Although there is general agreement that pQCD is able to make successful predictions for the exclusive processes at asymptotic limit ($Q^2 \to \infty$), the applicability of pQCD to these processes at experimentally available Q^2 region has been being debated and attracted much of attention. The difficulties in practical calculation mainly come from the end-point singularity, *i.e.* in the end-point region ($x \to 0, 1$ with x being the fractional momentum carried by the parton) the virtuality of intermediate states is small and the running couple constant α_s becomes large, thereby perturbation expansion might be illegal. However, perturbative calculation can be rescued with the help of some techniques to cure the end-point singularity [8-15], for example, the incorporation of the transverse structure of the pion wave function [8–10], the introduction of an effective gluon mass [11] and a frozen running coupling constant [11,12]. Recently, Li and Sterman [13,14] proposed a modified hard scattering approach (MHSA) for the hadronic form factor by taking into account the customarily neglected partonic transverse momentum as well as Sudakov corrections. They point out that pQCD calculation for the pion form factor begins to be self-consistent at about $Q \sim 20\Lambda_{QCD}$, which is similar to the conclusion given in [8]. More recently, Ji, Pang and Szczepaniak [15] arrived at a similar conclusion as [8,13, 14] by analyzing the factorization perturbation formalism for the pion form factor in the framework of light-cone time-order perturbative theory. These studies shed light on applying pQCD to exclusive processes at intermediate energy region.

However, there is still a crucial problem which has not been solved, that is although improved pQCD calculation for the exclusive processes is self-consistent at currently experimentally accessible energy region, the numerical predictions are generally far smaller than the experimental data. For example, pQCD prediction for the pion form factor is

$$F_\pi(Q^2 \to \infty) = 16\pi\alpha_s(Q^2)C_F \int [dx][dy]\phi(x)\frac{1}{x_2 y_2 Q^2}\phi(y) = \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}, \quad (1)$$

where $[dx] = dx_1 dx_2 \delta(1 - x_1 - x_2)$, $[dy] = dy_1 dy_2 \delta(1 - y_1 - y_2)$, $f_\pi = 93$ MeV is the pion decay constant, and $\phi(x)$ is the distribution amplitude of the pion meson. The asymptotic form for the distribution amplitude has been employed in obtaining the send expression in (1), since any distribution amplitudes for the pion meson should approach the asymptotic form as $Q^2 \to \infty$,

$$\phi^{(as)}(x) = \sqrt{3} f_\pi x_1 x_2. \quad (2)$$

Equation (1) gives only 1/3 of the experimental data at intermediate energy region. Although the Chernyak and Zhitnitsky (CZ) model for the distribution amplitude

$$\phi^{(CZ)}(x) = 5\sqrt{3} f_\pi x_1 x_2 (x_1 - x_2)^2 \quad (3)$$

may enhance the prediction for the pion form factor to the correct direction, the perturbative calculation with CZ distribution amplitude has been criticized seriously [6,7] because the nonperturbative end-point region is much emphasized in the CZ model. Recently studies on the pion-photon transition form factor [16] also show that the pion

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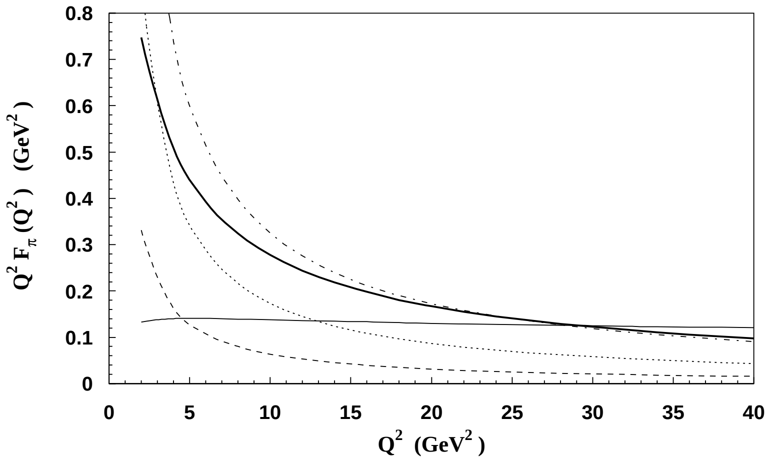

Fig. 1. Twist-2 and twist-3 contributions to the pion form factor. Each curve is explained in the text

distribution amplitude at currently experimentally available energy region is much like the asymptotic form but not the CZ form. Hence, how to match the perturbative calculation with the experimental data is an interesting issue. There are two possible explanations: one is that non-perturbative contributions will dominate in this region; the other is that non-leading order contributions in perturbative expansions may be also important in this region. To make choice between the two possible explanations one needs to analyze all of the important non-leading contributions carefully. These contributions come from higher-twist effects, higher order in α_s and higher Fock states *etc.*. Field, Gupta, Otto and Chang [17] pointed out that for the pion form factor the contribution from the next-leading order in α_s is about 20% $\sim$ 30%. Employing the modified hard scattering approach [13,14], [18] and [19] considered the transverse momentum effect in the wave function and found that the transverse momentum dependence in the wave function plays the role to suppress perturbative prediction. More recently, Tung and Li [20] reexamine the perturbative calculation for the pion form factor in the MHSA by respecting the evolution of the pion wave function in b (the transverse extent of the pion) and employing the two-loop running coupling constant in the Sudakov form factor. It is found [20] that the evolution of the pion wave function in b improves the match of perturbative prediction with the experimental data. However, in order to answer the question whether the perturbative calculation is able to make reliable prediction for the exclusive processes at currently experimentally available energy region, the other non-leading contributions such as that from higher twist effects and higher Fock states [2, 21] should also be analyzed carefully.

It has been expected that the power corrections to the pion form factor ($\sim 1/Q^4$) which come form the higher twist terms of the pion wave function may be important in the intermediate energy region [22–26] since there is a large factor $\sim m_\pi^2/m_0$ (m_π being the pion meson mass and m_0 being the mean u- and d-quark masses) in the twist-3 wave function. However, the calculations for these higher twist contributions are more difficult than that for the leading twist (twist-2) because of the end-point singularity becoming more serious. The leading twist wave functions in the initial and final states being proportional to $x_1 x_2$ (x_1 and x_2 being the fractional momenta carried by the quark and anti-quark) and $y_1 y_2$ (see (2)) may cancel the end-point divergent factor $1/x_2 y_2$ coming from the hard-scattering amplitude. However, the asymptotic behavior of twist-3 wave function is x- (y-)independent (see (20)), which has no help at all to cure the end-point singularity. In this case, Sudakov form factor is expected to be able to assure the reasonableness of the perturbative calculation. Unfortunately, the estimations for the twist-3 contribution in the medium energy region do not agree with each other [22–26] (see Fig. 1). [23] predicates that

$$F_\pi(Q^2) = \frac{16\pi\alpha_s(Q^2) f_\pi^2}{Q^2} \times \left\{1 + \frac{m_\pi^4}{Q^2 m_0^2} \alpha_s^{-8/9}(Q^2) J^2(Q^2)\right\} \quad (4)$$

with

$$J(Q^2) = \frac{1}{3}\left[\ln\ln(Q^2/\Lambda_{\rm QCD}^2) + a\right]. \quad (5)$$

The first and second terms in (4) correspond to the leading twist (twist-2) and next-to-leading twist (twist-3) contributions respectively. In [23] the double logarithmic (DL) corrections are calculated in the one loop approximation and it is supposed that the sum of all DL corrections transforms to the exponential function form (Sudakov form factor). Hence it is argued that the divergent factor $1/x_2 y_2$ at $x_2(y_2) \to 0$ is modified by the following way

$$\frac{1}{x_2 y_2} \to \frac{1}{x_2 y_2} \exp\left\{-[\alpha_{\rm s}({\rm Q}^2)/2\pi]{\rm C_F L}({\rm x_2, y_2, k_\perp, l_\perp})\right\}, \quad (6)$$

with

$$\begin{aligned} L(x_2, y_2, k_\perp, l_\perp) = {} & \ln({\rm Q}^2/{\rm k}_\perp^2)\ln(1/{\rm x}_2) - \ln^2(1/{\rm x}_2) \\ & + \ln({\rm Q}^2/{\rm l}_\perp^2)\ln(1/{\rm y}_2) - \ln^2(1/{\rm y}_2) \\ & - \ln(1/{\rm x}_2)\ln(1/{\rm y}_2)\,. \end{aligned} \quad (7)$$

It is argued [23] that the integral with function $L(x_2, y_2, k_\perp, l_\perp)$ can not be calculated unambiguously. This uncertainty is incorporated to the factor a being $1 \le a \le 2$

in the function $J(Q^2)$ (5). According to (4), the twist-3 contribution is larger than the asymptotic term (twist-2 contribution) in the region of $Q^2 \leq 30\,\mathrm{GeV}^2$. Reference [24] includes the Sudakov corrections in a similar way as [23] but improved the estimation on the function $J(Q^2)$ and the running mass $m(Q^2)$, and gives

$$F_\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2} \times \left[1 + \frac{m_\pi^4}{Q^2 m_0^2}\frac{\pi}{6\alpha_s(Q^2)}\left(\frac{\alpha_s(1\mathrm{GeV}^2)}{\alpha_s(Q^2)}\right)^{8/9}\right]. \quad (8)$$

It can be found from (8) that the twist-3 contribution is larger that the twist-2 contribution at about $Q^2 \leq 15\,\mathrm{GeV}^2$. Reference [25] analyzes the Sudakov effects by introducing an cut-off on the integral region instead of introducing the transverse momenta $k_\perp$ and $l_\perp$, and gives another prediction

$$F_\pi(Q^2) = \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2} \times \left[1 + \frac{m_\pi^4}{Q^2 m_0^2}\frac{1}{6}\left(\ln\frac{Q^2}{\Lambda_{\mathrm{QCD}}^2}\right)^{8/9}\right]. \quad (9)$$

Equation (9) tells us that the twist-3 contribution is about $2 \sim 0.6$ of the leading twist contribution at the energy region $2\,\mathrm{GeV}^2 \leq Q^2 \leq 10\,\mathrm{GeV}^2$. All of the above calculations (4), (8) and (9) give correct power suppression ($\sim 1/Q^2$) behavior for the twist-3 contribution in the large Q^2 region, but their predictions for the dependence on $\ln Q^2$ are very different. The main reason for these differences is that Sudakov corrections are evaluated in different approximations in [23], [24] and [25]. In the modified hard scattering approach for the exclusive processes proposed by Li and Sterman [13,14], the customarily neglected partonic transverse momentum are combined with Sudakov corrections, and the Sudakov form factor is expressed in a more convenient space (b-space), which provides an more reliable and systematical way to evaluate the Sudakov effect. Li and Sterman's formalism is originally obtained for studying the contribution from the leading twist wave function. We point out that for the pion electromagnetic form factor the MHSA can be extended to evaluate the contribution coming from the twist-3 terms of the pion wave function. One manifest advantage of MHSA is that there is no other phenomenological parameter but the input wave function need to be adjusted. The purpose of this work is to analyze the twist-3 wave function contribution to the pion form factor in the framework of MHSA.

2 Formalism

We first review the derivation of the modified hard-scattering formalism for the leading twist (twist-2) contribution to the pion form factor [13]. Taking into account the transverse momenta $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$ that flow from the wave functions through the hard scattering leads to a factorization form with two wave functions $\psi(x,\mathbf{k}_\perp)$ and $\psi(y,\mathbf{l}_\perp)$ corresponding to the external pions combined with a hard-scattering function $T_H(x,y,Q,\mathbf{k}_\perp,\mathbf{l}_\perp)$, which depends in general on transverse as well as longitudinal momenta,

$$F_\pi^{(t=2)}(Q^2) = \int [dx][dy]\int d^2\mathbf{k}_\perp d^2\mathbf{l}_\perp \psi(x,\mathbf{k}_\perp,P_1,\mu) \times T_H^{(t=2)}(x,y,Q,\mathbf{k}_\perp,\mathbf{l}_\perp,\mu)\psi(y,\mathbf{l}_\perp,P_2,\mu)\,, \quad (10)$$

where P_1 and P_2 are the momenta of the incoming and outgoing pion respectively, $Q^2 = 2P_1\cdot P_2$ and μ is the renormalization and factorization scale. To the lowest order in perturbation theory, the hard-scattering amplitude $T_H^{(t=2)}$ is to be calculated from one-gluon-exchange diagrams. Neglecting the transverse momentum dependence in the numerator of $T_H^{(t=2)}$ one can obtain,

$$T_H^{(t=2)}(x,y,Q,\mathbf{k}_\perp,\mathbf{l}_\perp,\mu) = \frac{16\pi C_F\alpha_s(\mu)x_2Q^2}{\left[x_2Q^2+\mathbf{k}_\perp{}^2\right]\left[x_2y_2Q^2+(\mathbf{k}_\perp-\mathbf{l}_\perp)^2\right]}\,, \quad (11)$$

where $C_F = 4/3$ is the color factor and $\alpha_s(\mu)$ is the QCD running coupling constant. The first and the second terms in the denominator come from fermion and gluon propagators respectively.

Equation (10) can be expressed in the **b**- and **h**-configurations via Fourier transformation

$$F_\pi^{(t=2)}(Q^2) = \int [dx][dy]\frac{d^2\mathbf{b}}{(2\pi)^2}\frac{d^2\mathbf{h}}{(2\pi)^2}\varphi(x,\mathbf{b},P_1,\mu) \times T_H^{(t=2)}(x,y,Q,\mathbf{b},\mathbf{h},\mu)\varphi(y,\mathbf{h},P_2,\mu)\,, \quad (12)$$

where wave functions $\varphi(x,\mathbf{b},P_1,\mu)$ and $\varphi(y,\mathbf{h},P_2,\mu)$ take into account an infinite summation of higher-order effects associated with the elastic scattering of the valence partons, which give Sudakov suppressions to the large-$b(h)$ and small-$x(y)$ regions [13,27,28],

$$\varphi(\xi,\mathbf{b},P,\mu) = \exp\left[-s(\xi,b,Q)-s(1-\xi,b,Q) -2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(g(\bar{\mu}))\right]\times\phi\left(\xi,\frac{1}{b}\right). \quad (13)$$

Here $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension. $s(\xi,b,Q)$ is Sudakov exponent factor [13,27,28],

$$\begin{aligned} s(\xi,b,Q) &= \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{-\hat{b}}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{-\hat{b}}-1\right) - \frac{A^{(1)}}{2\beta_1}(\hat{q}+\hat{b}) \\ &-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(-2\hat{b})+1}{-\hat{b}} - \frac{\ln(-2\hat{q})+1}{-\hat{q}}\right] \\ &-\left(\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln(\frac{1}{2}e^{2\gamma-1})\right)\ln\left(\frac{\hat{q}}{-\hat{b}}\right) \\ &+\frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q})-\ln^2(-2\hat{b})\right]\,, \end{aligned} \quad (14)$$

where

$$\begin{aligned}\hat{q} &= \ln[\xi Q/(\sqrt{2}\Lambda)], \quad \hat{b} = \ln(b\lambda),\\ \beta_1 &= \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24},\\ A^{(1)} &= \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{1}{3}\pi^2 - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln(\frac{1}{2}e^{\gamma}) \,. \end{aligned} \tag{15}$$

n_f is the number of quark flavors and γ is the Euler constant. In the derivation of (14), the one-loop running coupling constant has been employed. It is pointed out [29] that additional two terms will appear in the $s(\xi, b, Q)$ expression if the two-loop running coupling constant is used. The two terms reduce the prediction for the pion form factor by only a few percent [29] in the intermediate energy region. So for simplicity, we neglect these terms.

Applying the renormalization group equation to $T_H^{(t=2)}$ and substituting the explicit expression for $T_H^{(t=2)}$, one can obtain the following expression for the pion form factor

$$\begin{aligned}F_\pi^{(t=2)}(Q^2) = &\int [dx][dy] \int b\, db \int h\, dh\, 16\pi C_F \alpha_s(t) x_2 Q^2\\ &\times K_0(\sqrt{x_2}Qh)\phi(x,1/b)\phi(y,1/h)\\ &\times [\theta(b-h)K_0(\sqrt{x_2}Qb)I_0(\sqrt{y_2}Qh)\\ &+\theta(h-b)I_0(\sqrt{x_2}Qb)K_0(\sqrt{y_2}Qh)]\\ &\times \exp\left(-S(x,y,Q,b,h,t)\right),\end{aligned} \tag{16}$$

where

$$\begin{aligned}S(x,y,Q,b,h,t) = &\left[\left(\sum_{i=1}^{2} s(x_i,b,Q) + \sum_{i=1}^{2} s(y_i,Q,h)\right)\right.\\ &\left. - \frac{1}{\beta_1}\ln\frac{\hat{t}}{-\hat{b}} - \frac{1}{\beta_1}\ln\frac{\hat{t}}{-\hat{h}}\right].\end{aligned} \tag{17}$$

K_0 and I_0 are the modified Bessel functions of order zero. t is the largest mass scale appearing in $T_H^{(t=2)}$,

$$t = \max\left(\sqrt{xy}Q, 1/b, 1/h\right). \tag{18}$$

If b is small, radiative corrections will be small regardless of the values of x because of the small α_s. When b is large and xyQ^2 is small, radiative corrections are still large in $T_H^{(t=2)}$, but φ will suppress these regions. In (16), $\phi(x,1/b)$ and $\phi(y,1/h)$ are two input "wave functions" which respect the non-perturbative physics. In the large-Q^2 region, they can be taken as the asymptotic form of the twist-2 distribution amplitude (2) [13,27,28].

In the above discussion, only the leading twist wave function is considered. Now, we address the contributions coming form the twist-3 wave functions. The operators which contribute to the twist-3 parts of the pion wave function include γ_5 and $\gamma_5\sigma_{\mu\nu}$, and the two matrixes might mix under the consideration of the evolution equation for two-quark state in the pseudoscalar channel. It is pointed out in [23,24] that the twist-3 wave function of pion can be expressed as

$$\begin{aligned}&\psi^{(t=3)}(x,\mathbf{k}_\perp)\\ &\simeq \gamma_5\phi_3\left[1 - i\frac{2(x_1-x_2)}{Q^2}P_1^\mu\sigma_{\mu\nu}P_2^\nu - i\frac{x_1x_2}{\mathbf{k}_\perp^2}P_1^\mu\sigma_{\mu\nu}k_\perp^\nu\right],\end{aligned} \tag{19}$$

where $\mathbf{k}_\perp$ is the partonic transverse momentum. ϕ_3 is the distribution amplitude of twist-3 [22–25],

$$\phi_3 = \frac{f_\pi}{4\sqrt{n_c}}\frac{m_\pi^2}{\bar{m}(Q^2)}, \tag{20}$$

where $m_\pi = 139$ MeV is the pion meson mass and $\bar{m}(Q^2)$ is the mean value of the u- and d-quarks masses at the scale Q^2,

$$\bar{m}(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu_0^2)}\right)^{4/\beta_0} m_0(\mu_0^2), \tag{21}$$

with $\beta_0 = 11 - \frac{2}{3}n_f$, and $m_0(1\,\mathrm{GeV}^2) = 7 \pm 2\,\mathrm{MeV}$.

The hard scattering amplitude for the twist-3 wave function differs from that for the twist-2 wave function, which turns out to be

$$\begin{aligned}&T_H^{(t=3)}(x,y,Q,\mathbf{k}_\perp,\mathbf{l}_\perp,\mu)\\ &= \frac{64\pi C_F\alpha_s(\mu)x_2}{\left[x_2Q^2 + \mathbf{k}_\perp{}^2\right]\left[x_2y_2Q^2 + (\mathbf{k}_\perp - \mathbf{l}_\perp)^2\right]}\,.\end{aligned} \tag{22}$$

Following the derivation for the leading twist wave function we can obtain the twist-3 contribution to the pion form factor in the modified hard-scattering approach,

$$\begin{aligned}F_\pi^{(t=3)}(Q^2) = &\int [dx][dy] \int bdb \int hdh 64\pi C_F\alpha_s(t)x_2\\ &\times K_0(\sqrt{x_2}Qh)\phi_3(x)\phi_3(y)\exp\left(-S(x,y,Q,b,t)\right)\\ &\times [\theta(b-h)K_0(\sqrt{x_2}Qb)I_0(\sqrt{y_2}Qh)\\ &+\theta(h-b)I_0(\sqrt{x_2}Qb)K_0(\sqrt{y_2}Qh)]\,.\end{aligned} \tag{23}$$

It can be found that the hard scattering amplitudes $T_H^{(t=2)}$ (11) and $T_H^{(t=3)}$ (22) are divergent in the end-point region $x, y \to 0$, $k_\perp, l_\perp \to 0$. However, the twist-2 contribution to the pion form factor can be calculated readily because the twist-2 wave functions being proportional to x_1x_2 and y_1y_2 respectively cancel the divergent factor $1/x_2y_2$ in the $T_H^{(t=2)}$. Furthermore, the Sudakov corrections also suppress the contribution from the end point region. For the twist-3 contribution, the wave function is a constant in the whole region of x (see (20)), which has no help at all to cure the divergent factor $1/x_2y_2$ in the $T_H^{(t=3)}$. In this case, the Sudakov form factor may guarantee that the calculation is reliable since the factor e^{-S} rapidly decreases to zero more rapidly than any power of $x(y)$ at the end-point region (see (14) and (17)).

3 Numerical result and discussion

We present the numerical evaluations for the twist-2 and twist-3 contributions to the pion form factor in Fig. 1. The thinner solid curve is MHSA prediction for the twist-2 contribution (16). The thicker solid curve is twist-3 contribution to the pion form factor obtained in this work (23), while the dash-dotted is the result of [23] (the second term in (4) with $a = 1.5$). The dotted and dashed

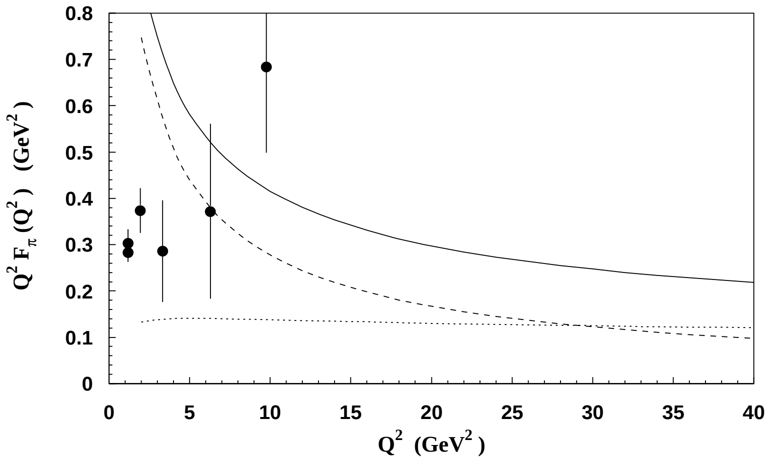

Fig. 2. Perturbative prediction for the pion form factor including both twist-2 (dotted curve) and twist-3 (dashed curve) contributions. The solid curve is the sum of twist-2 and twist-3 contributions. The data are taken from [30]

curves are the results given in [24] (the second term in (8)) and [25] (the second term in (9)) respectively. All of the calculations given in this work, [23], [24] and [25] show that compared with the leading twist contribution, the twist-3 contributions are suppressed by the factor $1/Q^2$ at asymptotic limit ($Q^2 \to \infty$). But the predictions are different in the medium and lower energy regions. Our result is much larger than the result of [25] in the energy region of $2\,\mathrm{GeV}^2 \leq Q^2 \leq 40\,\mathrm{GeV}^2$, and a little larger than the result of [24] as $Q^2 \geq 5\,\mathrm{GeV}^2$. Reference [23] and this work give very similar results in the large-Q^2 region, but our prediction is a little smaller than the result of [23] at about $Q^2 \leq 15\,\mathrm{GeV}^2$. The Sudakov corrections are respected systematically in MHSA, while they are evaluated in various approximations in [23–25], so the prediction in this work is more reliable. In Fig. 2, we include both twist-2 and twist-3 contributions (obtained in this work) to the pion form factor, and compare with the experimental data. The dotted and dashed curves are twist-2 and twist-3 contributions respectively, and the solid curve is the sum. Compared to the leading (twist-2) contribution, the twist-3 contribution is negligible at asymptotic limit since it is suppressed by the factor $1/Q^2$. However, the twist-3 contribution is comparable with and even larger than the leading twist contribution at intermediate region of Q^2 ($2\,\mathrm{GeV}^2 \leq Q^2 \leq 40\,\mathrm{GeV}^2$). Also it can be found that the perturbative calculations including both twist-2 and twist-3 contributions are larger than the experiment data at about $Q^2 \leq 5\,\mathrm{GeV}^2$. One can expects that the other nonleading contributions such as those coming form higher Fock states may be also important at lower energy regions.

In summary, we analyzed the twist-3 contribution to the pion electromagnetic form factor in the modified hard scattering approach in which Sudakov corrections are respected systematically, and compared with various approximate calculations. It is found that the twist-3 contribution is enhanced significantly since the twist-3 wave function is independent of the fractional momentum carried by the parton and has a large factor $\sim m_\pi^2/m_0$, while the twist-2 wave function is proportional to x_1x_2 (y_1y_2) which cancels the end-point divergent factor $1/x_2y_2$ in the hard-scattering amplitude. Thus, although it is suppressed by the factor $1/Q^2$ as compared with the leading (twist-2) contribution, the twist-3 contribution is comparable with and even large than the leading twist contribution at intermediate region of Q^2 being $2 \sim 40$ GeV^2. The perturbative predictions including both twist-2 and twist-3 contributions are larger than the experiment data at lower energy regions, which indicates the importance to study the other nonleading corrections at these energy regions.

Acknowledgements. This work partially supported by the Postdoc Science Foundation of China and the National Natural Science Foundation of China. F.G. Cao would like to thank professor B. V. Geshkenbein for providing more inferences for their work [23], and professor M. V. Terentyev for providing the preprint/ITEP-45 (1982).

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